SOME TOPICS ON HESSENBERG VARIETIES

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If X is a nonsingular projective variety of complex dimension d together with an embedding $X \hookrightarrow \mathbb{P}^N$, then the volume of the embedding is defined by

$$\mathrm{vol}(X \hookrightarrow \mathbb{P}^N) := \frac{1}{d!}\deg(X \hookrightarrow \mathbb{P}^N) = \frac{1}{d!}\int_X c_1(L)^d$$

where L is the very ample line bundle on X associated to the embedding. If X is a flag variety $\mathrm{Flags}(\mathbb{C}^n)$ together with a Plücker embedding, then its volume is known to agree with the volume of the associated Gelfand-Zetlin polytope.

Regular semisimple Hessenberg varieties are a certain family of nonsingular subvarieties of $\mathrm{Flags}(\mathbb{C}^n)$, so the Plücker embedding of $\mathrm{Flags}(\mathbb{C}^n)$ to $\mathbb{P}^N$ induces the embeddings of regular semisimple Hessenberg varieties to $\mathbb{P}^N$. I will talk about how to represent their volumes in terms of faces of the Gelfand-Zetlin polytope.

This is joint work with Megumi Harada, Tatsuya Horiguchi, and Seonjeong Park.

References

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[2] V. Kiritchenko, E. Smirnov, and V. Timorin, *Schubert calculus and Gelfand-Zetlin polytopes*, Russian Math. Surveys, 2012, 67 (4) 685-719; arXiv:1101.0278.